



Flash flood warning system based on fully dynamic hydrology modelling

G. Pejanovic, S. Petkovic, B. Cvetkovic
and S. Nickovic

*Republic Hydrometeorological Service
Belgrade, Serbia*

(South-East European Virtual Climate Change Center - SEEVCCC)

Numerical hydrologic modeling

- Hydrology modelling – one of most important component in global climate models to appropriately represent
 - hydrological cycle
 - energy fluxes in the Earth's atmosphere and at the surface
- Significant room for further improvements
- limited success in the past due to lack of adequate input data
- More recent improvements:
 - More accurate and high-resolution data on
 - topography, river routing, and soil types
 - Precipitation observations/predictions
- More recent improvements:
 - variety of hydrology modeling approaches:
 - simplified conceptual
 - kinematic methods
 - complex dynamic methods

- The most complex models include full dynamic governing equations
 - momentum equations, along with the equation of mass continuity, are used in their full extent.
 - Such approach
 - permits representation of hydrology scales ranging from flash floods to flows of large slow river watersheds.
 - do not need model calibration
 - could be unstable for vanishing surface water height if not appropriately numerically treated

Kinematic approximation: the the most used method in hydrology modelling

- Continuity equation - prognostic
- Momentum equations – diagnostic
 - Manning velocities calculates from the balance between the gradient and the friction slope forces
 - **Advantage**: no numerical instability
 - **Disadvantage**: simplified governing equations

$$u_M = \sqrt{\frac{h^{4/3}}{n^2 \sqrt{u^2 + v^2}} \frac{\partial h}{\partial x}}$$

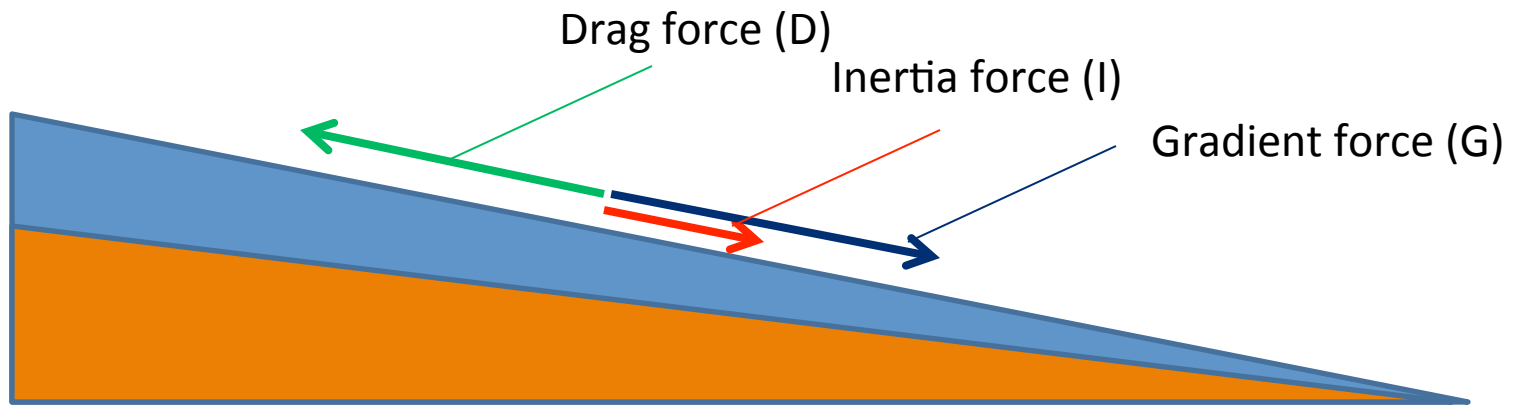
$$v_M = \sqrt{\frac{h^{4/3}}{n^2 \sqrt{u^2 + v^2}} \frac{\partial h}{\partial y}}$$

Full dynamics (FD) vs. kinematics (KN)

- FD model more accurate
- FD: friction slope term requires special treatment (Froude number <2)
- KN: simplifications avoids problem but simplifies the equations (Froude number >2)
- Most watershed models adapt KN approach
- KN cannot accurately represent large-scale, more inert processes

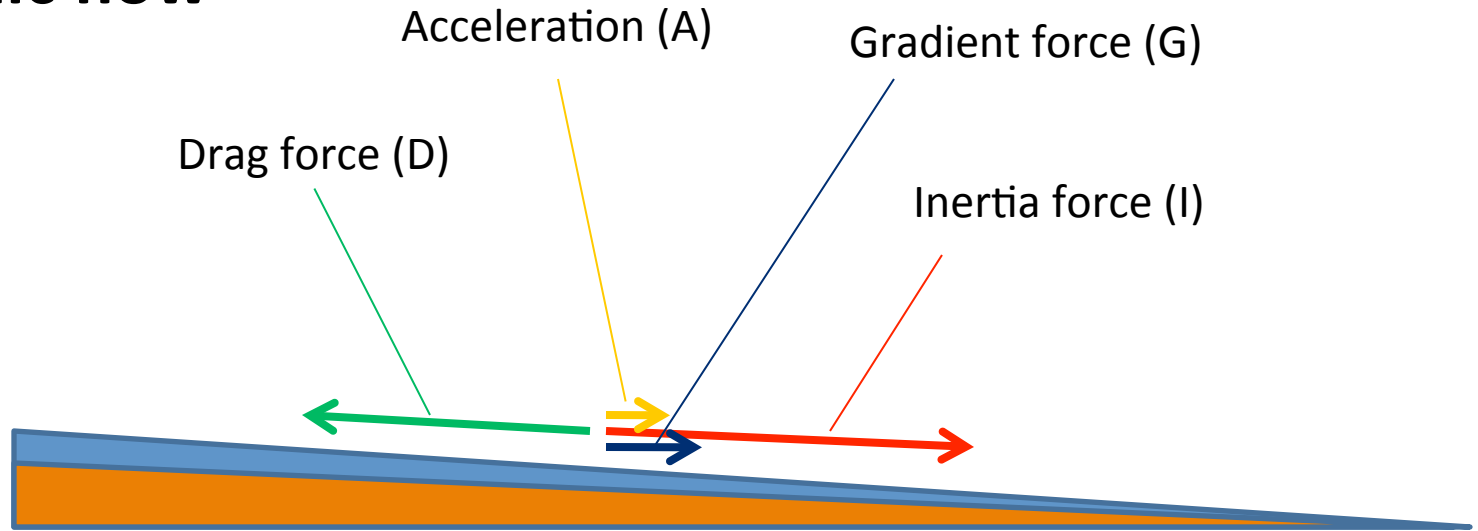
$$\text{Froude number}(Fr) = \frac{\text{Inertia forces}}{\text{Kinematic forces}} = \frac{V}{\sqrt{gh}}$$

Kinematic flow



- **steep** topography
- Gradient force dominant over inertia force (advection)
 - $\rightarrow Fr < 1$
- kinematic approximation $D \sim (I+G)$
- inappropriate for slow flows

Full dynamic flow



- weak topography slope
- Inertia force (advection) dominant over gradient force
- $\rightarrow Fr > 1$
- kinematic approximation $A \sim (I+G) - D$
- appropriate for both fast and slow flows

HYPROM (*) approach

- ❑ Aim: to avoid a kinematic or other restrictive approximation
- ❑ Dynamics based on the **Saint-Venant equations**
- ❑ Both momentum and continuity equations are prognostic

(*) Nickovic, S., G. Pejanovic, V. Djurdjevic, J. Roskar, and M. Vujadinovic (2010), HYPROM hydrology surface-runoff prognostic model, *Water Resour. Res.*, 46, W11506, doi:10.1029/2010WR009195

HYPROM - Full dynamic (FD) equation concept

$$\left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] + g \left[\frac{\partial h}{\partial x} + S_{fx} - S_{0x} \right] = 0$$

$$\left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] + g \left[\frac{\partial h}{\partial y} + S_{fy} - S_{0y} \right] = 0$$

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} + \dot{H} = 0$$

Friction slope terms

$$S_{fx} = \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}}$$

$$S_{fy} = \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}}$$



=0

Kinematic approximation

neglects inertia forces !! which we want to avoid



$=0$

Kinematic approx

Acceptable for **floods**
(where friction and gradient
forces are in approx.
balance)

Not appropriate for basins
with **slow flows**

→ **Full dynamic system
to be used**

HYPRON governing equations:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \left[\frac{\partial h}{\partial x} + S_{fx} - S_{0x} \right] = 0$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \left[\frac{\partial h}{\partial y} + S_{fy} - S_{0y} \right] = 0$$

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} + \dot{H} = 0$$

○ h - points
+ u,v - points

A-B-C-D-E-F river points

river routing

Novel components in HYPRON

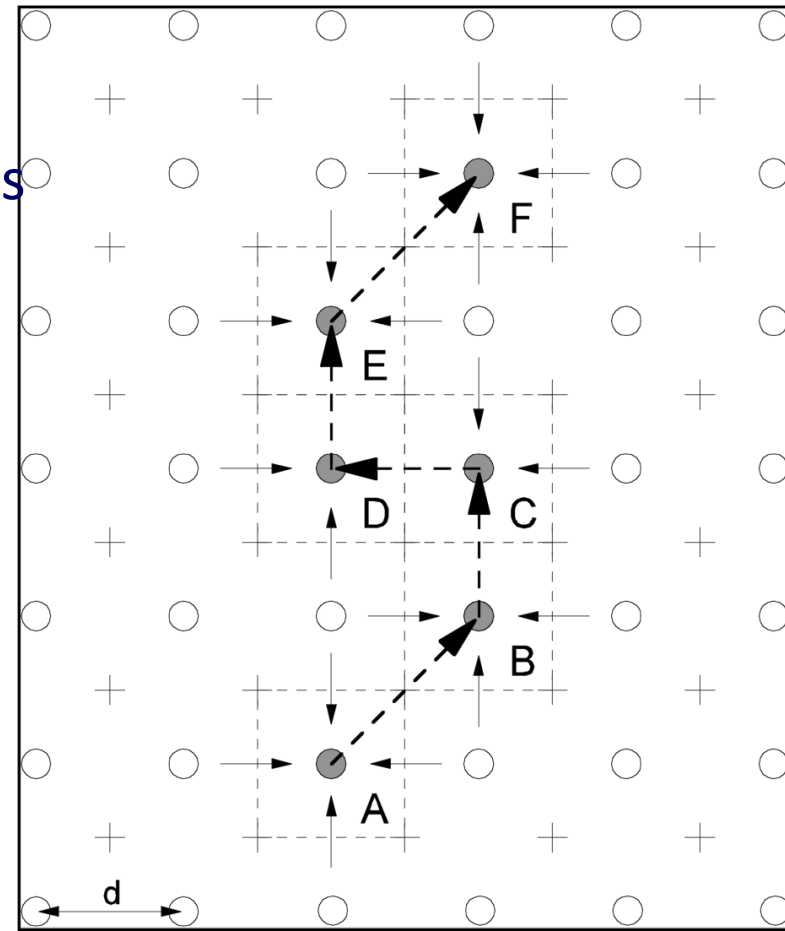
- **NO** approximation in the governing eq-s

- numerically **stable** numerics

- new numerical technique for **preventing grid decoupling** noise

- suitable for scales ranging from **local** (flash floods) to **climate** (large rivers, e.g. Danube)

- computationally **very efficient**



Full Dynamics approach requires completely different numerical approach to resolve the Instability due to vanishing water heights!

Friction slope

$$S_{fs} = \frac{n^2 \sqrt{u^2 + v^2}}{h^{4/3}} U$$



**Potential source of
model instability when $h \rightarrow 0$**

Fiedler and Ramirez (2002) suppress the instability by imposing an artificial threshold for the minimum water depth $h_c = 10^{-10}$ m

Friction slope numerics – HYPROM approach

Implicit time scheme applied

Unconditionally stable method

Convergent for $\Delta t \rightarrow 0$ $h \rightarrow 0$ $u, v \rightarrow 0$

$$u^{n+1} = \left[\frac{u - g \frac{\partial h}{\partial x} \Delta t}{1 + B \Delta t} \right]^n$$

More info in *Nickovic et al (2010)*

Friction slope numerics in HYPROM

[29] We propose instead an unconditionally stable integration scheme for the friction slope terms that is physically sound. For simplicity, let us consider a system composed of friction and height gradient terms,

$$\begin{aligned}\frac{\partial u}{\partial t} + \frac{gn^2\sqrt{u^2 + v^2}}{(\bar{h}^x)^{4/3}}u + g\delta_x h &= 0, \\ \frac{\partial v}{\partial t} + \frac{gn^2\sqrt{u^2 + v^2}}{(\bar{h}^y)^{4/3}}v + g\delta_y h &= 0.\end{aligned}\quad (10)$$

System (10) can be written in a simpler form as

$$\begin{aligned}\frac{\partial u}{\partial t} + B(u - u_M) &= 0, \\ \frac{\partial v}{\partial t} + B(v - v_M) &= 0,\end{aligned}\quad (11)$$

if we define the following parameters,

$$\begin{aligned}B &\equiv \frac{gn^2\sqrt{u^2 + v^2}}{(\bar{h}^x)^{4/3}}, \quad u_M = \frac{h^{4/3}}{n^2\sqrt{u^2 + v^2}} \frac{\partial h}{\partial x}, \\ v_M &= \frac{h^{4/3}}{n^2\sqrt{u^2 + v^2}} \frac{\partial h}{\partial y}\end{aligned}\quad (12)$$

Note that the equations in (11) are of a Newtonian-rudging type in which velocities are relaxed toward the Manning velocities u_M and v_M . We apply an implicit time scheme to (11) and obtain the following:

$$\begin{aligned}\frac{u^{n+1} - u^n}{\Delta t} + B^n u^{n+1} + g(\delta_x h)^n &= 0, \\ \frac{v^{n+1} - v^n}{\Delta t} + B^n v^{n+1} + g(\delta_y h)^n &= 0.\end{aligned}\quad (13)$$

By solving the equations for level $n + 1$, we finally get a numerical scheme of the following form,

$$\begin{aligned}u^{n+1} &= \left[\frac{u - g\Delta t\delta_x h}{1 + B\Delta t} \right]^n, \\ v^{n+1} &= \left[\frac{v - g\Delta t\delta_y h}{1 + B\Delta t} \right]^n.\end{aligned}\quad (14)$$

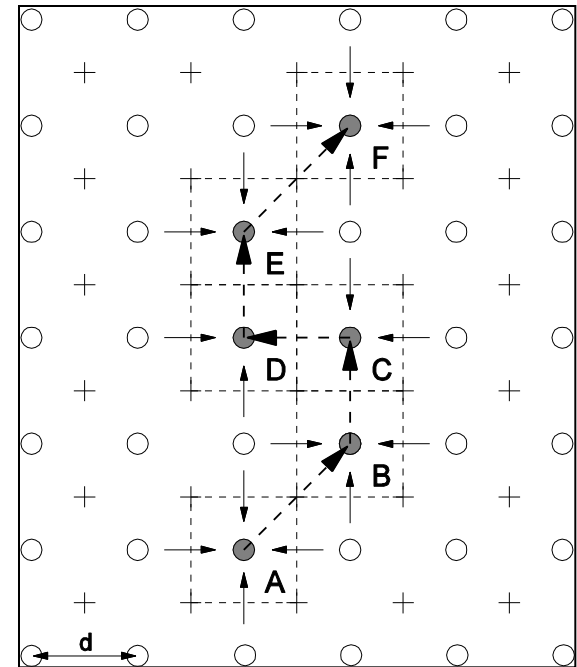
River routing

$$\frac{\partial U}{\partial t} + U \delta_s \bar{U}^s + g \delta_s (R + h_s) + \frac{n^2 |U|}{R^{4/3} s} U = 0$$

$$\frac{\partial R}{\partial t} + \delta_s (\bar{R}^s U) + \dot{R} = 0$$

s – river direction

- River – a water collector from surrounding points
- Same numerics as for non-river points



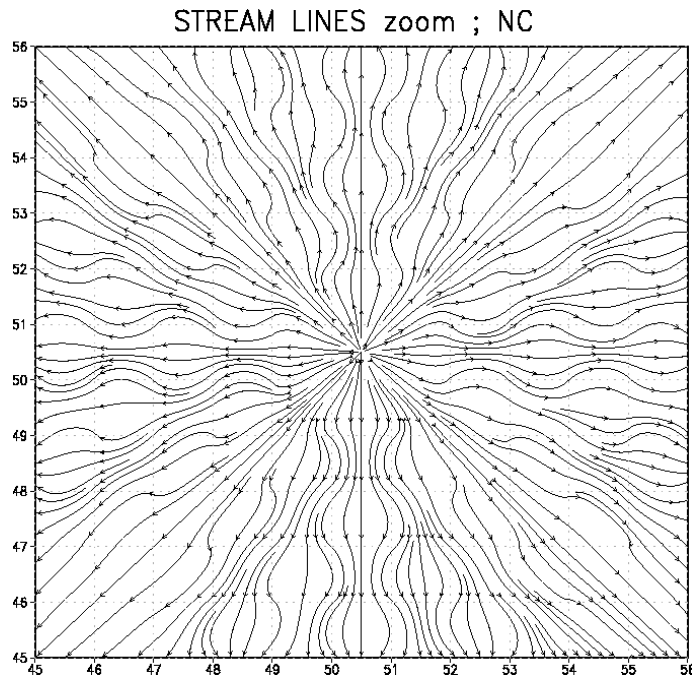
River path:
A-B-C-D-E-F

A numerical method proposed for the source term: averaging method which redistributes water mass to neighbouring points and avoids two-grid interval noise in the E or B horizontal grids

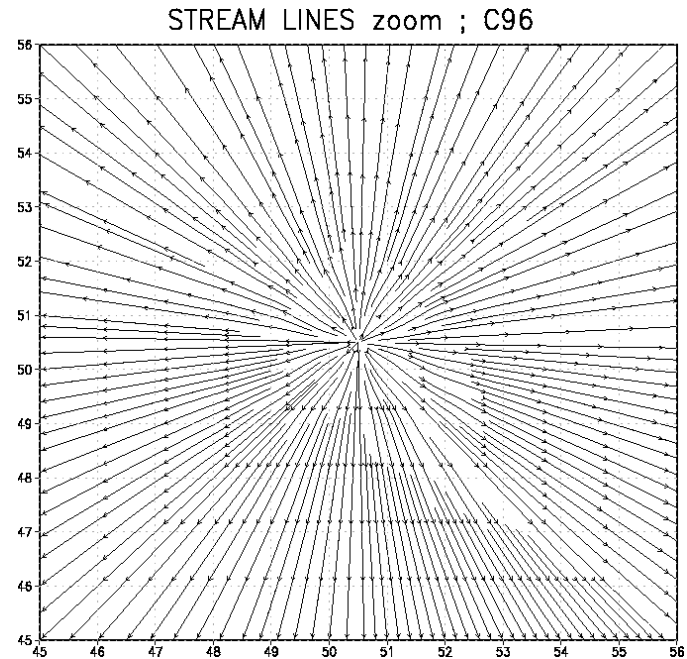
$$\frac{\partial h}{\partial t} + H(\delta_x u + \delta_y v) - \frac{1}{2} \left(\dot{H} + \overline{\dot{H}}^{xy} \right) = 0$$

Synthetic experiment with HYPROM

(1pt source forcing)

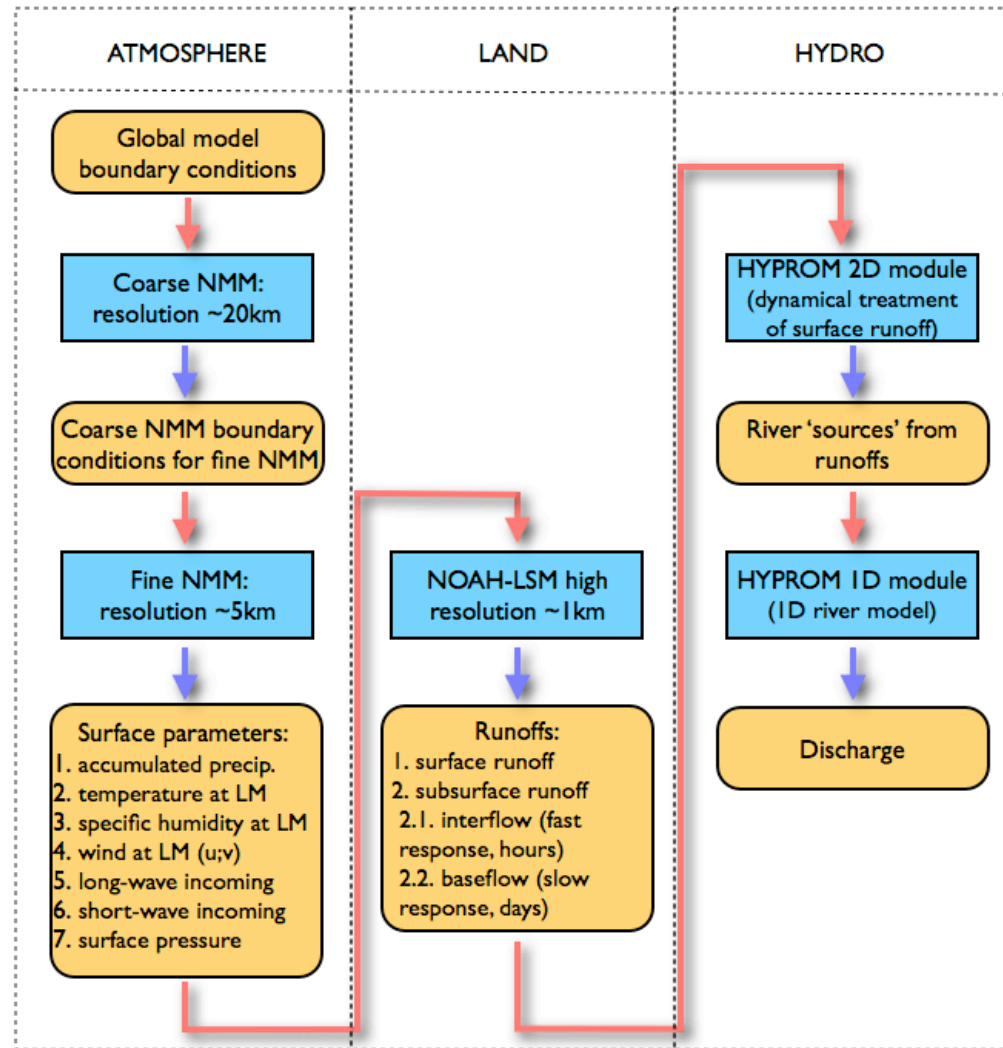


Without modification



With modification

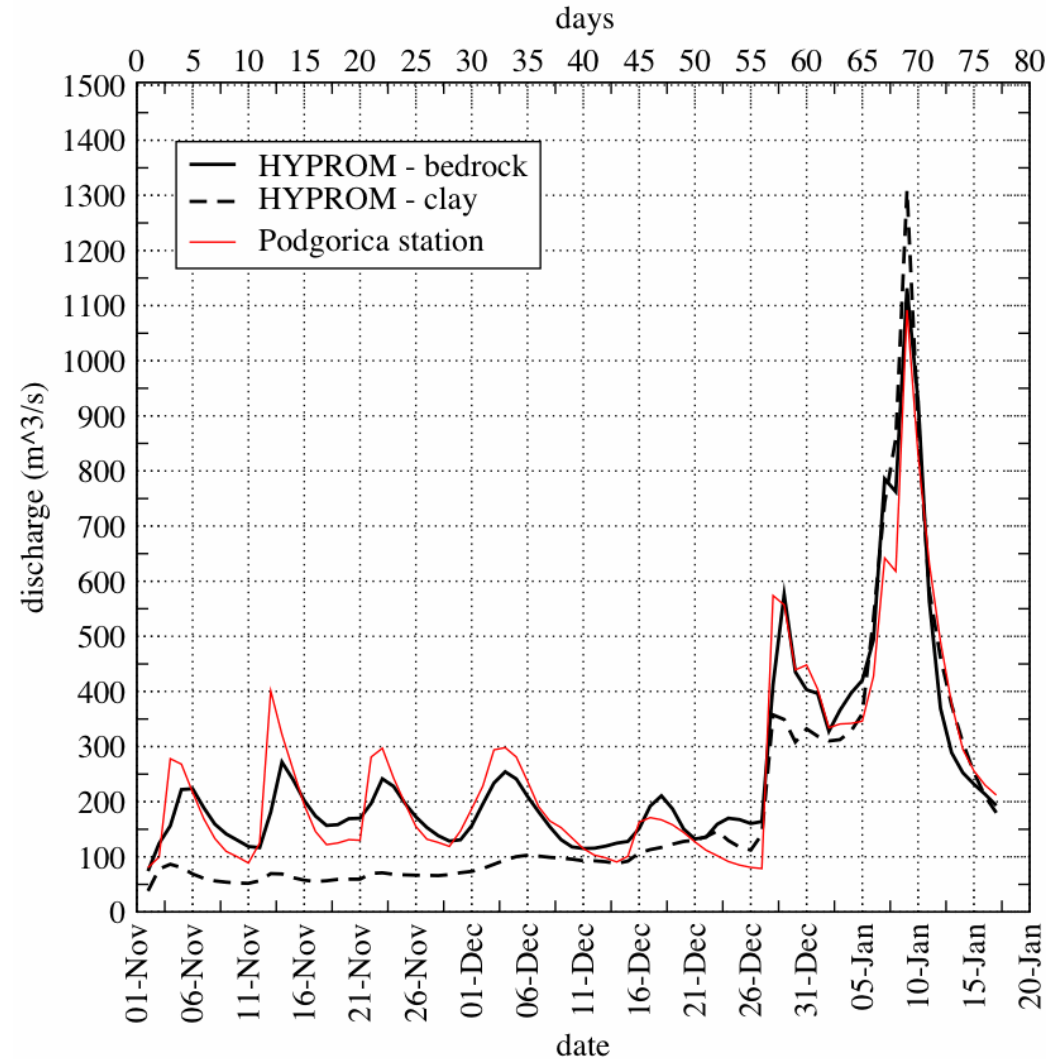
HYPROM integrated with the NCEP/NMM atmospheric model



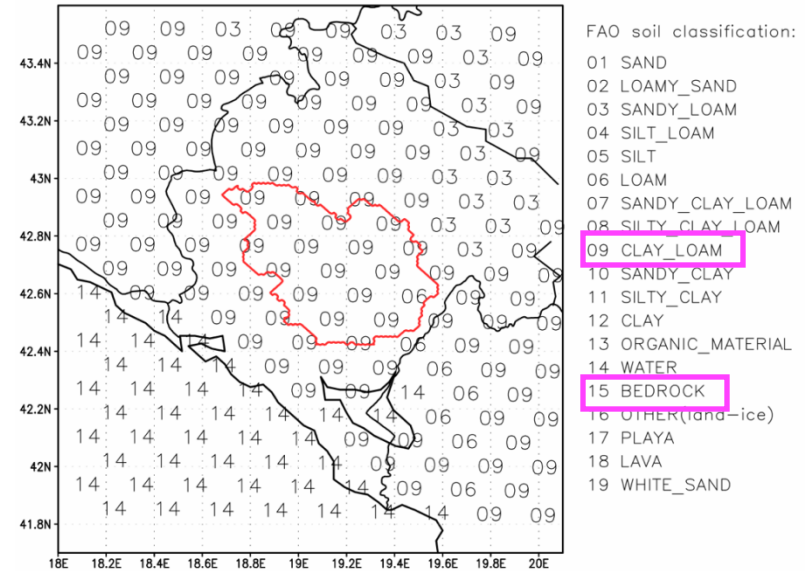
LM - lowest atmospheric model level

Moraca river - Sensitivity to soil type

Small catchment case



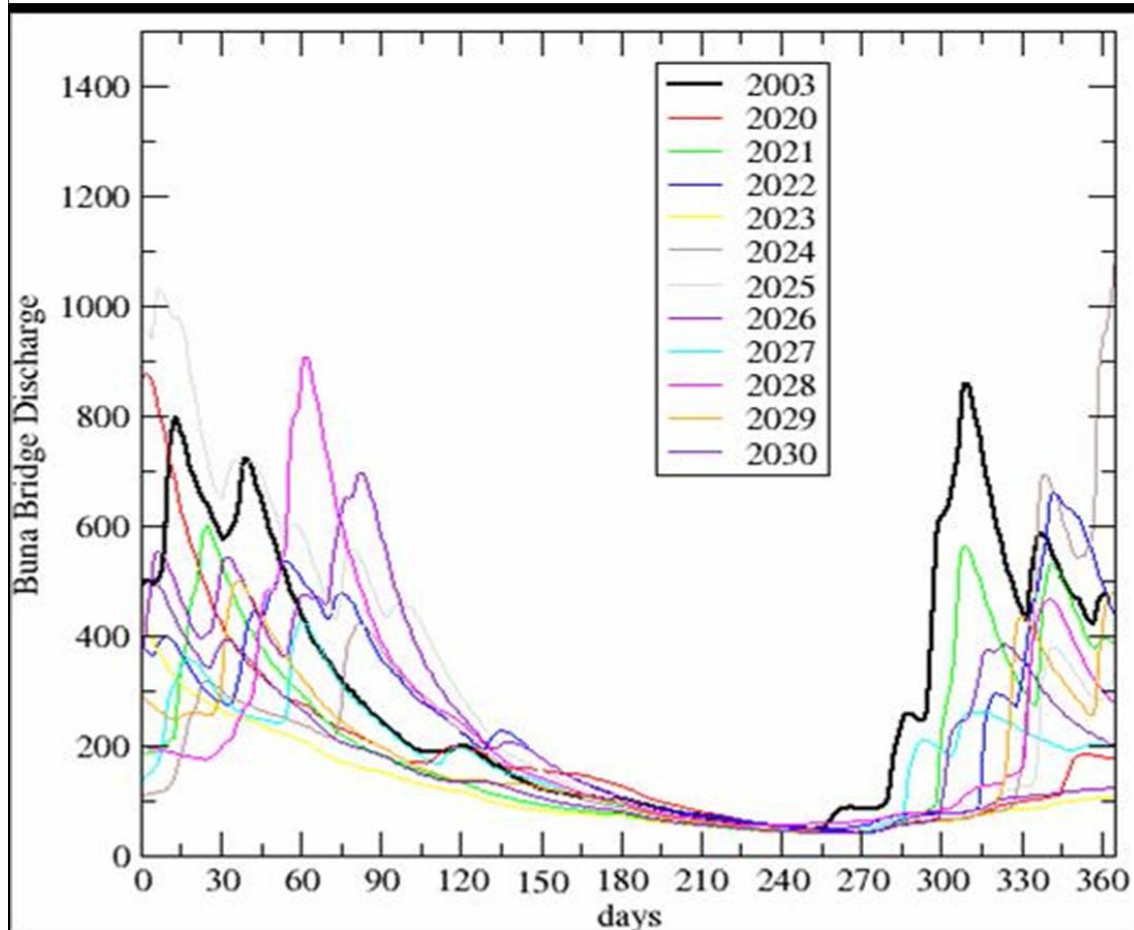
Soil type distribution over Montenegro



parameter	Clay Loam (09)	Bedrock (15)
sat. diffusivity	0.113×10^{-4}	0.136×10^{-3}
sat. conductivity	2.45×10^{-6}	1.41×10^{-4}
porosity	0.465	0.20
CH constant	8.17	2.79

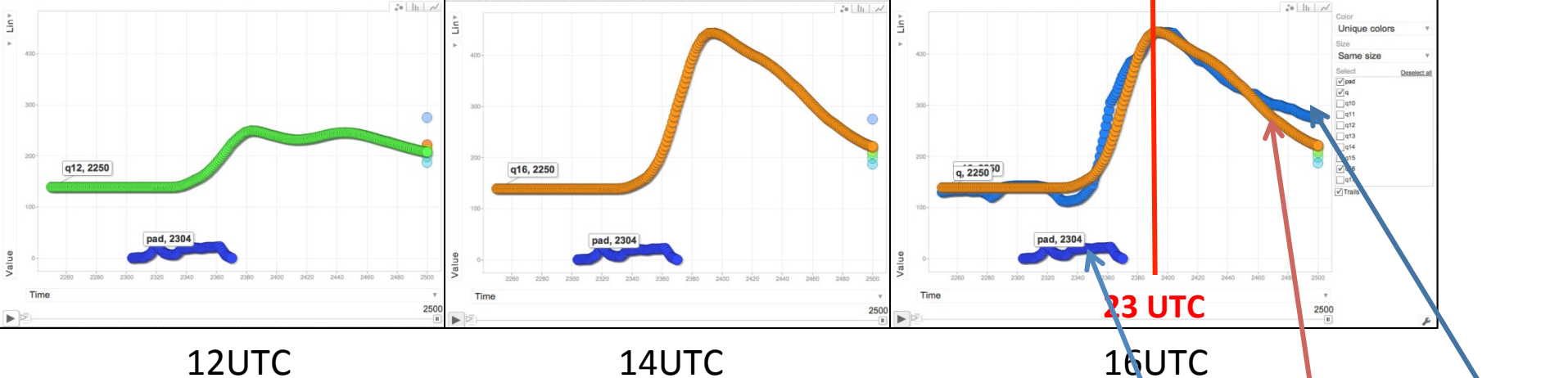
HYPROM and climate/seasonal assessments

Buna/Bojana river discharge (m^3/s) at Buna Bridge under the atmospheric conditions of the A1B scenario of IPCC for the period 2020-2030



Hydrology Prognostic Model

Djurdjevic et al, 2011



S. Morava River flash flood case

23 Jan 2015

disasterous consequences

HYPROM run

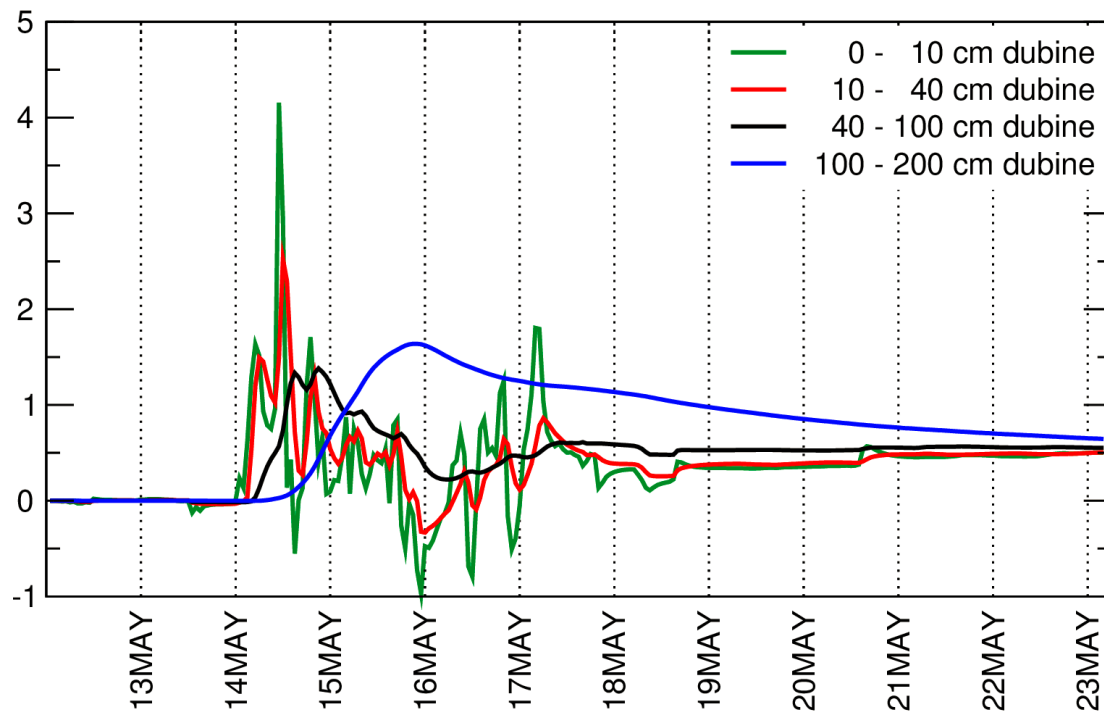
Driven by radar precipitation

Predicted correctly the max discharge 7
hours in advance



Most recent developments

- HYPROM dynamics has been fully coupled with the NCEP/ NMMB non-hydrostatic atmospheric model
- a two-way interaction (atmosphere-hydrology feedbacks)
(*Vujadinovic-Mandic, 2015; PhD Thesis*)



**Volumetric soil moisture difference
ctrl-feedback exp at 4 model soil levels**